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Major work on the dynamics of coupled rigid bodies was done. We studied both the case of three coupled rigid bodies in the plane, with a complete stability, bifurcation, and chaotic solutions analysis, but also studied the three dimensional case. In the energy-momentum method, for mechanical systems with Hamiltonian H of the form kinetic energy (K) plus potential (V), ~~we have found~~ a way to choose variables ~~in a way~~ that makes the determination of stability conditions sharper and more computable. ~~The~~ poisson brackets of free boundary fluid equations has been determined. In the homogeneous case, it has been shown already by Lewis, Marsden, Montgomery and Ratiu that the structure of the bracket is that of a Yang/Mills theory for the principal bundle whose total space consists of the embeddings of a given domain in space, the base space is the space of unparametrized fluid shapes and the group os the particle relabeling group. the geometric reasons for the integrability of the planar three point vortex motion is terms of dual pairs appearing in the study of the geometry of Poisson manifolds has been given by Adams and Ratiu in the paper cited at the beginning of this report. The study of the hydrodynamic bifurcations was begun on the example of a rigidly rotating incompressible homogeneous disk by Lewis, Marsden, and Ratiu in 1987.


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The following publications relevant to this proposal have appeared in the last year:

The dynamics of two coupled rigid bodies (J.E. Marsden, R. Grossman and P.S. Krishnaprasad), *Dynamical Systems Approaches to Nonlinear Problems in Systems and Circuits*, (ed by Salam and Levi), SIAM(1988)373-378.

The dynamics of coupled planar rigid bodies I. Reduction, equilibria, and stability, (Y.G. Oh, S. Sreenath, P.S. Krishnaprasad, and J.E. Marsden) *Dyn. Stab. of Systems*.3(1988) 25-49.

Lie Poisson integrators and Lie Poisson Hamilton Jacobi theory, (Ge Zhong and J.E. Marsden) *Physics Letters A.*, **113**(1988)134-139.

The Hamiltonian structure of nonlinear elasticity: The material and convective representations of solids, rods, and plates, (J.C. Simo, J.E. Marsden, and P.S. Krishnaprasad) *Arch. Rat. Mech. An.* **104**(1988)125-183.

The dynamics of coupled planar rigid bodies II. Bifurcations, Periodic solutions and chaos (S. Sreenath, Y.G. Oh, P.S. Krishnaprasad and J.E. Marsden) . *J. Dyn. and Diff. Eqns.* **1** (1989) 269-298.

Cartan Hannay Berry phases and symmetry (J.E. Marsden, R. Montgomery and T.S. Ratiu), *Cont. Math. AMS* **97**, 279-296

Block diagonalization and the energy momentum method, (J.E. Marsden, D.R. Lewis, T. Posbergh and J.C. Simo) *Cont. Math. AMS* **97**, 297-314

Stability analysis of a rigid body with attached geometrically nonlinear rod by the energy-momentum method (J.E. Marsden, T. Posbergh and J.C. Simo) *Cont. Math. AMS* **97**, 371-398.

Integrability and the 3 vortex problem (M. Adams and T. Ratiu), *Cont. Math. A.M.S.* **81**(1988).

Controlling Homoclinic Orbits (A.M. Bloch and J.E. Marsden), *Theor. and Comp. Fluid Mech.* **1**, 179-190.

In addition, the paper *Stability of Coupled Rigid Body and Geometrically Exact Rods: Block Diagonalization and the Energy Momentum Method*, has been accepted by *Physics Reports* and the papers *Stability of Relative Equilibria Parts I and II* have been submitted to the *Archive for Rational Mechanics and Analysis*. The paper of Ratiu and Mazur on *Stability of self gravitating disks* will appear in *Contemporary Mathematics*, and one on the *Poisson bracket for barotropic flow* will appear in the *Journal of Geometry and Physics*.

## *Summary of Accomplishments*

**Coupled Rigid Bodies.** Major work on the dynamics of coupled rigid bodies was done. We studied both the case of three coupled rigid bodies in the plane, with a complete stability, bifurcation, and chaotic solutions analysis, but also studied the three dimensional case. One of the supported students completed a stability and bifurcation analysis of the case of two coupled symmetric rigid bodies, and implemented a technique for the integration and visualization of their dynamics using the symplectic integrator ideas of Channell, Scovel, Ge and Marsden. Future work (with Bloch and Krishnaprasad) has now begun on the control theory of coupled rigid bodies. It will use the geometry already developed and the work of Marsden, Montgomery, and Ratiu on geometric phases to give attitude control.

**Block Diagonalization.** In the energy-momentum method, for mechanical systems with Hamiltonian $H$ of the form kinetic energy (K) plus potential (V), we have found a way to choose variables in a way that makes the determination of stability conditions sharper and more computable. These variables *separate the rotational and the vibrational modes in an effective way.* The setting is that of a mechanical system with symmetry (even rotational symmetry is not trivial!) and an associated momentum map $J$. In this set of variables (with the conservation of momentum constraint and a gauge symmetry constraint imposed) the second variation of the *augmented* Hamiltonian $H_\xi = H - \langle J, \xi \rangle$ block diagonalizes; schematically

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$$\delta^2 H_\xi = \begin{bmatrix} \begin{bmatrix} 2\times 2 \text{ rigid} \\ \text{body block} \end{bmatrix} & 0 \\ 0 & \begin{bmatrix} \text{Internal vibration} \\ \text{block} \end{bmatrix} \end{bmatrix}.$$

Furthermore, the internal vibrational block takes the form

$$\begin{bmatrix} \text{Internal vibration} \\ \text{block} \end{bmatrix} = \begin{bmatrix} \delta^2 V_\mu & 0 \\ 0 & \delta^2 K_\xi \end{bmatrix}$$

where $V_\mu = V + \frac{1}{2}\langle \mu, \mu \rangle$, $\langle\ ,\ \rangle$ being the (locked inertia) metric on the dual of the Lie algebra induced by the given kinetic energy, $\mu$ is the total conserved momentum of the system, $K_\xi(q, p) = \frac{1}{2} \| p - A_\xi \|^2$ and $A_\xi(q)$ is the metric flat (or the Legendre transform) of $\xi_Q(q)$. Here $\delta^2 K_\xi > 0$ so formal stability is equivalent to $\delta^2 V_\mu > 0$ and positive definiteness of the rigid body block, which separates out the overall rigid body motions from the internal motions of the system under consideration (for a geometrically exact rod, this includes shear and torsion).

The dynamics of the internal vibrations (such as the elastic wave speeds) depend on the rotational angular velocity. That is, the internal vibrational block is $\xi$-dependent, but in a way we can explicitly calculate. On the other hand, these two types of motions do not *dynamically decouple*, since the symplectic form does *not* block diagonalize. However, we can compute the off-diagonal terms explicitly (they turn out to be momentum maps that play a crucial role in how the block diagonalizing variables are constructed in the first place!) which determines the dynamic coupling.

These techniques have been applied to specific rotating elastic structures, both rods and three dimensional elastic structures.

**Poisson Brackets.** [The Poisson bracket of free boundary fluid equations has been determined. In the homogeneous case, it has been shown already by Lewis, Marsden, Montgomery and Ratiu that the structure of the bracket is that of a Yang-Mills theory for the principal bundle whose total space consists of the embeddings of a given domain in space, the base space is the space of unparamtrized fluid shapes and the group is the particle relabeling group.] For the compressible case, this structure is linked to the semidirect product theory familiar from the fixed boundary case. A general framework for these theories was developed in "The Hamiltonian structure of continuum mechanics in material, inverse material, spatial and convective representations (D.D. Holm, J.E. Marsden and T.S. Ratiu), *Séminaire de Mathématiques*

*supérieurs, Les Presses de L'Université de Montrèal,* **100**, (1986), 11-122. In the future we will be applying the block diagonalization method to this type of system.

**Three point vortex motion.** The geometric reasons for the integrability of the planar three point vortex motion in terms of dual pairs appearing in the study of the geometry of Poisson manifolds has been given by Adams and Ratiu in the paper cited at the beginning of this report.

**The Liquid Drop.** The study of the hydrodynamic bifurcations was begun on the example of a rigidly rotating incompressible homogeneous disk by Lewis, Marsden, and Ratiu in 1987 (Stability and bifurcation of a rotating liquid drop, *J. Math. Phys.* **28** (1987) 2508-2515). A nonlinear stability threshold was established and a bifurcation analysis has been carried out if this stability threshold is violated. The result states that a stable branch of relative equilibria, which look ellipse-like, emanates at the critical value of the parameter (the angular velocity or the angular momentum). This example has been a crucial ingredient into our recent work on the development of a bifurcation theory for Hamiltonian systems with symmetry. We will be investigating this theory in the next grant proposal.